**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans = 0.2676

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1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans** **= B is False** To calculate the number of employees under the age of 30 we need to find the proportion of employes with less age than 30 using the normal distribution. We can find the Z-score for 30.

**A is True** Since the Z-score for 44 is positive and Z-score 38 is 0 the proportion of employes older than 44 is greater than the proportion b/w 38 and 44. Therefore statement A is true

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1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans = 2 *X*1**

**Distribution** : If X1 follows a normal dist N(μ, σ2) will follow a normal distribution as well but it will be scaled by a factor of 2.

**Parameters**: The mean(μ) of 2 *X*1 will be 2 μ and the variance (σ2) will be 4 σ2.

So, 2 *X*1 ~ N(2 μ, 4 σ2)

Ans = X1+X2

**Distribution =** The sum of two independent normal random variables is also a normal random variable. if X1 ~N (μ, σ2 ) and X2 ~N (μ, σ2 ) and follow they are independent their sum X1+X2 will also follow a normal distribution.

**Parameters:** The mean of X1+X2 is the sum of the means of X1 and X2 which is μ+ μ=2 μ. The variance of X1+X2 is the sum of the variance of X1 and X2 which is σ2+ σ2= 2 σ2.

So, X1+X2 ~N(2 μ, 2 σ2)

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1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans =**  **Option D: 48.5, 151.5**

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1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans = A**

1.Calculate the mean and variance of the total profit:

* Total mean: Mean (Profit1) + Mean (Profit2) = 5+7 = 12Million dollars.
* Total variance = Variance (Profit1) + Variance (Profit2) = 9 +16 = 25 Million dollars2.

2. Convert to rupees using the conversion factor:

- Total Mean: 12\*45 = 540 million Rupees

- Total Standard deviation = sqrt(25)\*45 = 75 million rupees

3. To find 95% probability range centred mean you can use the Z-score for a 95% confidence lnterval. A 95% interval corresponds to 2 std. dev

- Lower bound: Total mean-2\*Total Standard deviation = 540-2\*75 = 390 million rupees

- Upper bound: Total mean+2\*Total Standard deviation = 540+2\*75 = 690 million rupees

**Ans = B**

1. The Z-score for the 5th percentile using standard normal distribution table is approximately is -1.645.
2. Calculating the 5th percentile profit in rupees

5th Percentile(Rupees): Total mean +(-1.645)\*Total Standard deviation = 540-1.645\*75 = 418.75 million rupees

**Ans = c**

1. Calculate the Z-score for zero profit for each division

Z-score for profit = (0-5)/sqrt (9) = -5/3

Z-score for profit = (0-7)/sqrt(16) = -7/4

1. Using a standard normal dist

Probability of Profit 1 making a loss = P(Z < -5/3)

Probability of Profit 2 making a loss = P(Z < -7/3)